ORC 67-40
JULY 1967

MAXIMIZATION OF SYSTEM RELIABILITY WITH LIMITED RESOURCES

by

Lawrence D. Bodin

OPERATIONS RESEARCH CENTER

COLLEGE OF ENGINEERING

UNIVERSITY OF CALIFORNIA - BERKELEY

24

MAXIMIZATION OF SYSTEM

RELIABILITY WITH LIMITED RESOURCES

by

Lawrence D. Bodin†
Operations Research Center
University of California, Berkeley

July 1967 ORC 67-40

This research has been partially supported by the Office of Naval Research under Contract Nonr-3656(18), the National Science Foundation under Grant GK-1684, the National Science Foundation under Grant GP-7417, and the U.S. Army Research Office-Durham, Contract DA-31-124-ARO-D-331 with the University of California. Reproduction in whole or in part is permitted for any purpose of the United States Government.

†Dr. Bodin is presently a staff member of the IBM Washington Scientific Center, Wheaton, Maryland.

ACKNOWLEDGMENT

I should like to thank Professor Richard E. Barlow of the Department of Industrial Engineering and Operations Research of the University of California, Berkeley, for his assistance and encouragement.

ABSTRACT

The optimization of system reliability of a series parallel system containing t types of components is found where the cost of purchasing the components is disregarded, a component can be assigned to more than one component position, and a limited supply of components is available for assignment. The optimal solution is found by ranking the reliabilities of the components of each type and searching over these ranks in the orders specified in this paper.

MAXIMIZATION OF SYSTEM RELIABILITY WITH LIMITED RESOURCES

by

Lawrence D. Bodin

1.0 INTRODUCTION

A series parallel arrangement of component positions has n subsystems connected in parallel where the i^{th} subsystem contains k_i component positions joined in series. In this system the $\sum_{i=1}^{n} k_i$ component positions are divided into t types--for example, resistor component positions, transistor component positions, capacitor component positions, etc. A sufficient number of components of each type with not necessarily identical reliabilities are available for assignment so that a feasible assignment of components to component positions can be made. Both the components and the system are subject to one type of failure and the other properties of coherent structures (Barlow and Proschan [1], Birnbaum, Esary, and Saunders [2], Esary and Proschan [3]). The problem is to determine that assignment of components to component positions which maximize the system reliability.

This analysis is applicable in the following type of situation. A manufacturer wishes to produce 100,000 identical series parallel systems each of which is made up of 10 transistors and 15 resistors. From various vendors, the manufacturer can purchase 300,000 transistors with reliability .9, 400,000 transistors with reliability .7, 300,000 transistors with reliability .3, 600,000 transistors with reliability .8, and 900,000 transistors with reliability .4. Thus, each system the manufacturer produces will contain three transistors of reliability .9, four transistors of reliability .7, three transistors of reliability .3, six resistors of reliability .8, and nine resistors of reliability .4. Knowing that a system with a higher reliability commands a higher price, the manufacturer wishes to assign components to component positions in such a way that the reliability of each system is identical and maximal.

2.0 Two Path Analysis

A two path series parallel system with path lengths k_1 and k_2 is given. Let ℓ_{ij} denote the number of component positions of type j on path $j = 1, 2, \ldots, t$. The components of type j available for assignment have reliabilities :

$$p_{1j} \geq p_{2j} \geq \cdots \geq p_{\ell_{1j} + \ell_{2j},\, j}, \; j = 1, 2, \ldots, t \; . \tag{1}$$

For convenience, the number of components of type j available for assignment have been taken equal to the number of component positions of that type. It is shown in Theorem (7) that if there are more components of type j available for assignment than is needed, we need only consider the $\ell_{1j} + \ell_{2j}$ most reliable components.

The following notation is utilized in the ensuing devr˙ɔpment:

- A_i, B_i - assignments of components to sockets.
- $h_{A_i}\left(p^{A_i}\right) = h_{A_i}$ = reliability of structure under assignment A_i .
- $h_{A_i} \vee h_{A_j} = h_{A_i} + h_{A_j} - h_{A_i} h_{A_j}$
- A_{ijk} - the set of components of type k assigned to path j under assignment A_i .
- $h_{A_{ijk}} = \prod_{c_r \in A_{ijk}} p_{rk}$. If $A_{ijk} = E$, the empty set, then $h_{A_{ijk}} \equiv 1$.
- $A_{ij} = \bigcup_{k=1}^{t} A_{ijk}$ - the set of components assigned to path j under assignment A_i .

The analysis of this section is based on the following three assignments:

- A_1 - components with reliabilities $p_{1j}, p_{2j}, \ldots, p_{\ell_{1j},\, j}$, $j = 1, 2, \ldots, t$, assigned to path 1 and the other components to path 2.

- A_2 - components with reliabilities $p_{1j}, \ldots, p_{\ell_{2j}, j}$, $j = 1, 2, \ldots, t$, assigned to path 2 and the other components to path 1.
- A_3 - an arbitrary assignment.

<u>Lemma 1</u>:

$$h_{A_3} \leq \max\left(h_{A_1}, h_{A_2}\right)$$

<u>Proof</u>:

For $i = 1, 2,$ define the following substructure assignments:

Q_i = set of components common to assignments A_1, A_2, A_3 on path i.

$Q_i = A_{1i}\, A_{2i}\, A_{3i}$.

R_i = set of components on path i common to both A_1 and A_3 but not to A_2 . $R_i = A_{1i}\, A_{3i}\, A_{2,\, 3-i}$.

S_i = set of components on path i under A_3 and path $3-i$ under A_1 .

$S_i = A_{3i}\, A_{1,\, 3-i}$.

T_i = set of components on path i common to both A_2 and A_3 but not to A_1 . $T_i = A_{2i}\, A_{3i}\, A_{1,\, 3-i}$.

U_i = set of components on path i under A_3 and path $3-i$ under A_2 .

$U_i = A_{3i}\, A_{2,\, 3-i}$.

The substructures associated with each of these substructure assignments is a series system with independent components; hence, the reliability of each substructure-- $h_{Q_i}, h_{R_i}, h_{S_i}, h_{T_i}, h_{U_i}$ --is the product of the reliabilities of the components in the substructure.

$$h_{A_1} = h_{A_{11}} \vee h_{A_{12}} = h_{Q_1} h_{R_1} h_{S_2} \vee h_{Q_2} h_{R_2} h_{S_1}$$

$$h_{A_2} = h_{A_{21}} \vee h_{A_{22}} = h_{Q_1} h_{T_1} h_{U_2} \vee h_{Q_2} h_{T_2} h_{U_1} \tag{2}$$

$$h_{A_3} = h_{A_{31}} \vee h_{A_{32}} = h_{Q_1} h_{R_1} h_{S_1} \vee h_{Q_2} h_{R_2} h_{S_2} = h_{Q_1} h_{T_1} h_{U_1} \vee h_{Q_2} h_{T_2} h_{U_2} .$$

Then

$$h_{A_3} - h_{A_1} = \left(h_{Q_2} h_{R_2} - h_{Q_1} h_{R_1}\right)\left(h_{S_2} - h_{S_1}\right)$$

$$h_{A_3} - h_{A_2} = \left(h_{Q_1} h_{T_1} - h_{Q_2} h_{T_2}\right)\left(h_{U_1} - h_{U_2}\right). \tag{3}$$

Since the number of components on path 1 (path 2) is fixed and Q_1 and R_1 (Q_2 and R_2) are on path 1 (path 2) under assignments A_1 and A_3, S_1 and S_2 have the same number of components m_1 . Hence, $h_{S_2} > h_{S_1}$. By the same argument, the number of components of type j, $j = 1, 2, \ldots, t$, on each path is fixed. Hence, the number of components of type j, m_{1j}, in S_1 and S_2 is fixed where $\sum_{j=1}^{t} m_{1j} = m_1$. Similarly, U_1 and U_2 have the same number of components m_2 so that $h_{U_1} \geq h_{U_2}$ and the number of components of type j, m_{2j}, in U_1 and U_2 is fixed also where $\sum_{j=1}^{t} m_{2j} = m_2$. Define the following sets: $R_{ij} \subset R_i$, $S_{ij} \subset S_i$, $T_{ij} \subset T_i$, $U_{ij} \subset U_i$ where i refers to path number and j refers to component type, $j = 1, 2, \ldots, t$, $i = 1, 2$.

If the lemma is false, then $h_{A_3} > \max\left(h_{A_1}, h_{A_2}\right)$. Thus, from (5.3), it can be concluded that

$$h_{Q_2} h_{R_2} - h_{Q_1} h_{R_1} > 0$$

$$h_{Q_1} h_{T_1} - h_{Q_2} h_{T_2} > 0 , \tag{4}$$

Implying

$$h_{R_2} h_{T_1} = \prod_{j=1}^{t} h_{R_{2j}} h_{T_{1j}} > \prod_{j=1}^{t} \left(h_{R_{1j}} h_{T_{2j}} \right) = h_{R_1} h_{T_2} . \tag{5}$$

If $R_{1j} \subset S_{1j}$ and $T_{1j} \subseteq U_{1j}$ have ℓ_{1j} components while $R_{2j} \subset S_{2j}$ and $T_{2j} \subset U_{2j}$ have ℓ_{2j} components, then $R_{2j} \subset T_{1j}$ and $R_{1j} \subset T_{2j}$ have $\ell_{1j} + \ell_{2j} - m_{1j} - m_{2j}$ components. Let v_{1j} and v_{2j} be the number of components of type j on path 1 and 2 and assume without loss of generality that $v_{1j} \leq v_{2j}$. Then, R_{1j} and $T_{2j} \subseteq A_{11}$ and R_{2j} and $T_{1j} \subseteq A_{21}$ where $p_{v_{1j}j} \geq p_{(v_{2j}+1)j}$. Hence, $h_{R_{2j}} h_{T_{1j}} \leq h_{R_{1j}} h_{T_{2j}}$ for component type j . Thus, inequality (5) is violated and we obtain a contradiction. ||

A_1 and A_2 are the only assignments that must be considered in determinimg the optimal assignment to a two path series parallel system. Theorem 2 proves that the system reliability under A_1 is greater (less) than the system reliability under A_2 if the product of the reliabilities of the components in A_{11} is greater (less) than the product of the reliabilities of the components in A_{22} .

<u>Theorem 2</u>:

(i) If $h_{A_{11}} > h_{A_{22}}$, then $h_{A_1} > h_{A_2}$.

(ii) If $h_{A_{11}} < h_{A_{22}}$, then $h_{A_1} < h_{A_2}$.

(iii) If $h_{A_{11}} = h_{A_{22}}$, then $h_{A_1} = h_{A_2}$.

<u>Proof</u>:

$$h_{A_1} - h_{A_2} = h_{A_{11}} + h_{A_{12}} - h_{A_{21}} - h_{A_{22}} . \tag{6}$$

Since $h_{A_{11}} h_{A_{12}} = h_{A_{21}} h_{A_{22}}$,

$$\frac{h_{A_{11}}}{h_{A_{22}}} = \frac{h_{A_{21}}}{h_{A_{12}}} \tag{7}$$

so that

$$\frac{h_{A_{11}} - h_{A_{22}}}{h_{A_{22}}} = \frac{h_{A_{21}} - h_{A_{12}}}{h_{A_{12}}} \tag{8}$$

Thus,

$$\frac{h_{A_{11}} - h_{A_{22}}}{h_{A_{21}} - h_{A_{12}}} = \frac{h_{A_{22}}}{h_{A_{12}}} > 1 \tag{9}$$

<u>Case I</u>:

If $h_{A_{11}} > h_{A_{22}}$, then $h_{A_{21}} > h_{A_{12}}$ and $h_{A_{11}} - h_{A_{22}} > h_{A_{21}} - h_{A_{12}}$. Hence from (6), $h_{A_1} > h_{A_2}$. This prove (i).

<u>Case II</u>:

If $h_{A_{11}} < h_{A_{22}}$, then $h_{A_{12}}$ and $h_{A_{11}} - h_{A_{22}} < h_{A_{21}} - h_{A_{12}}$. Hence, from (9) and (6) $h_{A_1} < h_{A_2}$. This proves (ii).

<u>Case III</u>:

If $h_{A_{11}} = h_{A_{22}}$, then $h_{A_{21}} = h_{A_{12}}$ so that $h_{A_1} = h_{A_2}$. ||

<u>Corollary 3.</u>

If $\ell_{1j} \leq \ell_{2j}$, $j = 1, 2, \ldots, t$, then $h_{A_1} \geq h_{A_2}$.

Proof:

Under the above hypothesis, $h_{A_{11}} \geq h_{A_{22}}$ so that either (i) or (iii) of Theorem 2 is applicable. $||$

Thus, if $\ell_{1j} \leq \ell_{2j}$, $j = 1, 2, \ldots, t$, the most reliable path possible to form is the path containing the smallest number of component positions with the most reliable components of each type assigned to these component positions. Moreover, when there is one type of component (t=1), Corollary 3 is always applicable and the optimal assignment is to assign the most reliable components to the path containing the least number of component positions.

3.0 n Path Analysis

Recall that ℓ_{ij} denote the number of component positions of type j on path i, j=1,2, ...,t, i=1,2, ..., n and the components of type j available for assignment have reliabilities

$$p_{1j} \geq p_{2j} \geq \cdots \geq p_{L_n(j)} \tag{10}$$

where $L_i(j) = \sum_{r=1}^{i} \ell_{rj}$, i = 0, 1, ..., n, and $L_o(j) = 0$. The following lemma is useful in designing a procedure for optimizing the reliability of the n path system.

Lemma 4:

If $\ell_{ij} \leq \ell_{i+1,\, j}$, i = 1, 2, ..., n-1, j = 1, 2, ..., t, then the optimal assignment B_1 is to assign $p_{(L_{i-1}(j)+1),j}$, $p_{(L_{i-1}(j)+2),j}$, ..., $p_{L_i j}$ on path i, j = 1, 2, ..., t, i = 1, 2, ..., n .

Proof:

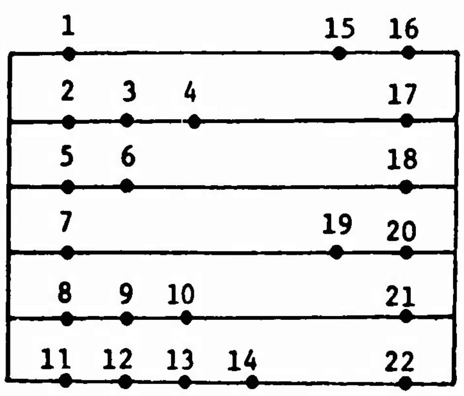
Under B_1 the assignment to any two path substructure satisfies Corollary 3 Let B_2 be an assignment distinct from B_1, that is to say, there exist a two path substructure such that under B_2, Corollary 3 is not satisfied. Let A_1 be the assignment to this two path substructure before applying Corollary 3 and A_i be the assignment after applying Corollary 3 . Then $h_{A_1} < h_{A_2}$. Hence

$$h_{B_2} = 1 - \left(1 - h_{A_3}\right)\left(1 - h_{A_2}\right) < 1 - \left(1 - h_{A_3}\right)\left(1 - h_{A_1}\right) = h_{B_3} \tag{11}$$

where A_3 is the assignment under B_2 to all paths not in A_2 and B_3 is the assignment under A_3 and A_1 . Thus, B_2 is not optimal and so a contradiction has been found. ||

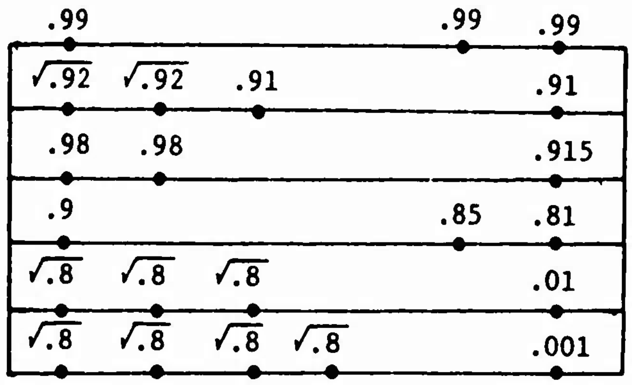
The procedure given in Lemma 4 initially finds for an n path system, the most reliable path assignment (the path with $\sum_{j=1}^{t} \ell_{1j}$ component positions). It then considers a new system with $n-1$ paths and repeats the above operation. Thus, at each step in the procedure, the most reliable path assignment with the components yet unassigned is found. This assignment procedure is reversible; i. e. the same assignment can be derived by forming at each step the least reliable path possible to form with the components yet unassignment. Example 1 shows that the procedure alluded to in Lemma 4 and the above discussion does not derive the optimal assignment for all n path series parallel systems with more than one type of component.

Example 1:

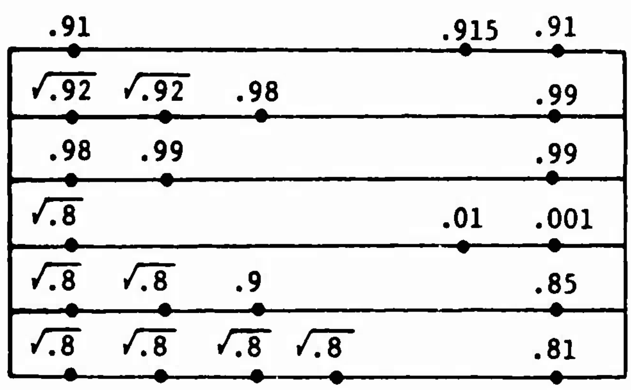


- Component positions 1 - 14 are of Type 1.
- Component positions 15 - 22 are of Type 2.
- Components of Type 1 have reliabilities .99, .98, .98, $\sqrt{.92}$, $\sqrt{.92}$, .91, .90, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$.
- Components of Type 2 have reliabilities .99, .99, .915, .91, .85, .81, .01, .001 .

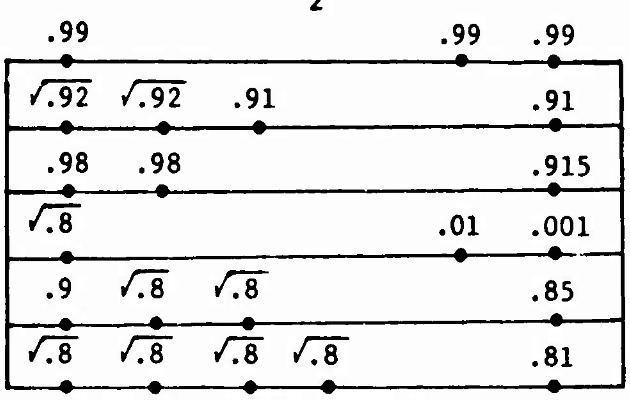
Three different assignments A_1, A_2, A_3 are given below. The selection criterion for A_1 is to find the most reliable path assignment possible to form with the components yet unassigned and make that assignment. The selection rule for A_2 is to find the least reliable path assignment possible to form with the components yet unassigned and make the assignment. The selection rule for A_3 is to use the selection rule for A_1 on the first three steps and then the selection rule for A_2 on the last three steps. The reliabilities are $h_{A_1} = .99973497$, $h_{A_2} = .9998079$, and $h_{A_3} = .9998977$ respectively.



A_1



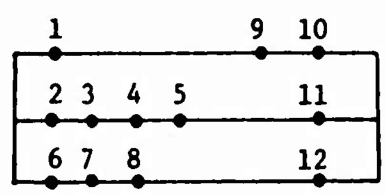
A_2



A_3

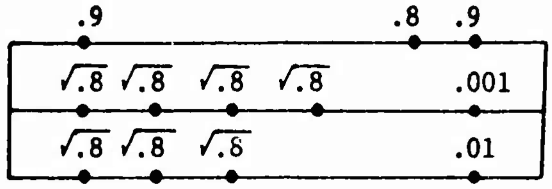
It is shown in Lemma 5 and Theorem 6 that for any n path series parallel system the optimal assignment is contained in the set of assignments $\mathcal{E}_n$ to the n path system generated by either selecting the most reliable or least reliable path assignment in the structure yet to be assigned and making that assignment. For a two path system, this result is proved in Theorem 2 Moreover, for a two path system, $\mathcal{E}_2$ contains but one element, the optimal solution. Thus, for an n path system, only 2^{n-2} assignments need be considered if no ties are encountered. If a tie is found, however, both assignments must be considered since a tie cannot be broken arbitrarily (see Example 2).

<u>Example 2</u>:

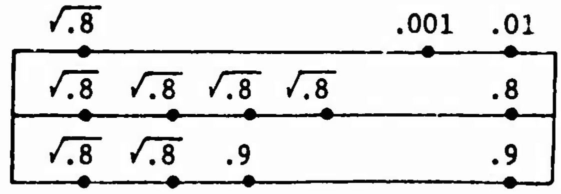


- Component positions 1 - 8 are of Type 1.
- Component positions 9 - 12 are of Type 2.
- Components of Type 1 have reliabilities .9, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$, $\sqrt{.8}$.
- Components of Type 2 have reliabilities .9, .8, .01, .001 .

By finding the maximum reliability path on the first step a tie occurs between paths 1 and 3. If both assignments are completed, two different assignments A_1 and A_2 are formed. $h_{A_1} = .660$ while $h_{A_2} = .828$.



A_1



A_2

Lemma 5:

If, for an assignment A_1 to an $n(\geq 3)$ path series parallel system, there exists a $2 \leq k \leq n-1$ path substructure whose assignment B_1 is not optimal to this k path substructure (this means there exists another assignment B_2 of the components assigned to this k path substructure which gives a greater reliability), then A_1 is not optimal to the n path system.

Proof:

Since $h_{B_1} < h_{B_2}$, $h_{A_1} = 1 - \left(1 - h_{B_3}\right)\left(1 - h_{B_1}\right) < 1 - \left(1 - h_{B_3}\right)\left(1 - h_{B_2}\right)$

where B_3 is the assignment under A_1 to all paths not in B_1. This proves the lemma. ||

Theorem 6:

The optimal assignment A^* to an n path series parallel system is a member of $\mathcal{E}_n$.

Proof:

By induction:

(i) For $n = 2$, it follows from Theorem 2.

(ii) Assume we are given an assignment A^* such that the assignment to all possible substructures with k paths $k = 2,3, \ldots, n-1$, is optimal (Lemma 5) and, hence, contained in its appropriate $\mathcal{E}_k$ (the induction hypothesis). We wish to show that $A^* \in \mathcal{E}_n$.

Since each substructure of K path is a member of its particular $\mathcal{E}_k$, the order of assigning the paths in the substructure is known (although not necessarily unique). The components assigned at each step in the assignment procedure to each substructure dominate all unassigned components to that substructure in the sense that they have the largest or smallest reliabilities depending upon whether a maximization or minimization operation is carried out. Let

- D_1 = assignment of components to the substructure made up of paths 1, 3, 4, ..., n .
- D_2 = assignment of components to the substructure made up of paths 2, 3, 4, ..., n .

By the induction hypothesis, since D_1 and D_2 are in their appropriate $\mathcal{E}_{n-1}$ and the order of assigning components to the paths under D_1 and D_2 and to any corresponding substructure is known, the order of assigning components to the paths under D_1 and D_2 are the same until either path 1 is encountered in D_1 or path 2 is met is D_2 . Let

- $F_1 = \{j|\text{path } j \text{ is assigned before path 1 in } D_1 \text{ and before path 2 in } D_2\}$.
- $F_2 = \{j|\text{path } j \text{ is assigned after path 1 in } D_1 \text{ and before path 2 in } D_2\} \cup \{1, 2\}$.
- $F_3 = \{j|\text{path } j \text{ is assigned after path 1 in } D_1 \text{ and after path 2 in } D_2\}$.

Without loss of generality assume that path 1 is encountered first.

<u>Case 1</u>:

If either F_1 or F_3 is not empty, the set of components available for assignment is partitioned into two or three parts--those assigned to the paths in F_1 , those assigned to the paths in F_2 , and those assigned to the paths in F_3 . All components assigned to the paths in F_1 have reliabilities which are either greater than or less than the components assigned to the paths in F_2 and F_3 and all components assigned to the paths in F_2 have reliabilities which are either greater than or less than the components assigned to the paths in F_3. By the induction hypothesis, the assignment of the components to the paths in F_1, F_2, and F_3 are optimal and the order of assigning the paths to each of the substructures is known. Hence $A^* \in \mathcal{E}_n$ and the order of assignment to A^* is to first assign the components to the paths in F_1 following the optimal order, then to the paths in F_2 following the optimal order and finally to F_3 following the optimal order.

Case 2:

If both F_1 and F_3 are empty, the $\Gamma_2 = \{1, 2, \ldots, n\}$. Let path k be the last path assigned to substructure 1, 3, 4, ..., n following the optimal order. Therefore, path k is the next to last path and path 2 is the last path assigned to substructure 2, 3, ..., n. However, path 2 can be assigned before path k in the substructure 2, 3, ..., n and the reliability is unaffected since in a two path substructure it makes no difference which path is assigned first as long as we change the maximization (minimizations) operation to a minimization (maximization) operation on the first step. Hence, if we redefine $F_2 = \{1, 2, \ldots, n\} - \{k\}$, $F_3 = \{k\}$, we are in case 1. This completes the induction.

Theorem 7:

If there exist m_i' components of type i available for assignment to an n path series parallel system and m_i sockets $(m_i' \geq m_i)$, then the optimal assignment uses the m_i most reliable components.

Proof:

For type i, let $p_1 \geq p_2 \geq \cdots \geq p_{m_i} \geq p_{m_i+1} \geq \cdots \geq p_{m_i'}$ be the components of type i available for assignment. Let A_1 be the optimal assignment in $\mathcal{E}_n$ using components $p_1, p_2, \ldots, p_{m_i}$, A_2 be the optimal assignment in $\mathcal{E}_n$ using an arbitrary subset of the m_i' components and A_3 be the assignment which follows the order of A_2 but uses components having reliabilities $p_1, p_2, \ldots, p_m$. Then $h_{A_1} \geq h_{A_3}$ and $h_{A_2} - h_{A_3} = \prod_{i=1}^{n} (1 - a_i') - \prod_{i=1}^{n} (1 - a_i)$ where $a_i(a_i')$ is the reliability of the i'th path under assignment $A_2(A_3)$. Since $a_i' \geq a_i$ for each i, $h_{A_3} \geq h_{A_2}$. This proves the theorem. ||

The easiest way to compute all members of $\mathcal{E}_n$ is to construct a tree and enumerate all possible cases. At each step in the process, two possible alternatives arise--either to maximize or minimize. An efficient way to carry out the enumeration is to utilize a last in - first out (LIFO) rule. Thus, if the first assignment is to maximize at each step, the next assignment under LIFO is to maximize at each step except the last, the third assignment is to maximize all but the next to the last path and so forth. The tree diagram for a five path system is given in Figure 1. Since the last two steps in the generation of any assignment in $\mathcal{E}_n$ can be found by Theorem 2, each path in the tree requires but three arcs.

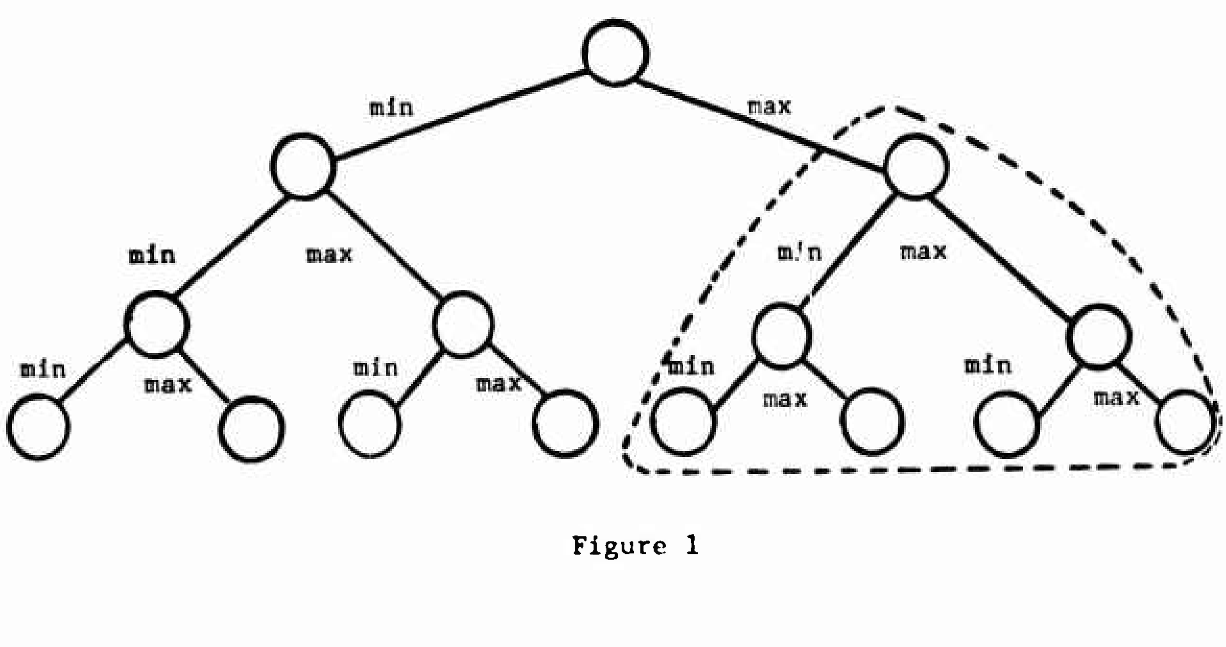


Figure 1

In certain instances, a complete enumeration is not necessary. Suppose that in carrying out the enumeration to the five path system pictured in Figure 1, path 1 is assigned first by maximizing and the sockets on the remaining four paths have the property that $\ell_{ij} \leq \ell_{i+1,j}$, $i = 2,3, \ldots, n-1$, $j = 1,2, \ldots, t$. Then Lemma 4 can be utilized and the entire subtree replaced by a single branch. This subtree is denoted by the dotted circle in Figure 1.

4.0 More General Structures

A set of series parallel systems connected in series are given and the sockets in one series parallel system are different from any other series parallel system in the sense that the components to be assigned to one system are independent of those in the other systems. Then the following theorem holds.

Theorem 8:

Under the above assumption, the assignment which maximizes the system reliability can be found by maximizing the reliability of each series parallel subsystem.

Proof:

Let $h_{(i)}$ denote the reliability of the i'th subsystem. Then the reliability of the entire system h_s is

$$h_s = h_{(1)} h_{(2)} \cdots h_{(m)} \tag{12}$$

where m is the number of subsystems. Since each subsystem's assignment is independent of the assignment to any other subsystem

$$\begin{aligned} \text{Max } h_s &= \text{Max } (h_{(1)} \cdots h_{(m)}) \\ &= \text{Max } h_{(1)} \text{ Max } h_{(2)} \cdots \text{Max } h_{(m)} . \end{aligned} \tag{13}$$

This proves the theorem. ||

Thus, h_s is found by m applications of Theorem 6 .

BIBLIOGRAPHY

1. Barlow, R. E. and F. Proschan, MATHEMATICAL THEORY OF RELIABILITY, John Wiley and Sons, New York, (1965).

2. Birnbaum, Z. W., J. D. Esary, and S. C. Saunders, "Multi-Component Systems and Structures and Their Reliability," Technometrics, Vol. 3, No. 1, pp. 55-77, (1961).

3. Esary, J. D. and F. Proschan, "Coherent Structures of Nonidentical Components," Technometrics, Vol. 5, No. 2, pp. 183-189, (1963).

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of California, Berkeley	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

MAXIMIZATION OF SYSTEM RELIABILITY WITH LIMITED RESOURCES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Research Report

5. AUTHOR(S) (Last name, first name, initial)

BODIN, Lawrence D.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July 1967	18	3

8a. CONTRACT OR GRANT NO. Nonr-3656(18) b. PROJECT NO. NR 042 238 c. Research Project No.: WW 041 d.	9a. ORIGINATOR'S REPORT NUMBER(S) ORC 67-40 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
SEE TITLE PAGE	MATHEMATICAL SCIENCE DIVISION

13. ABSTRACT

The optimization of system reliability of a series parallel system containing t types of components is found where the cost of purchasing the components is disregarded, a component can be assigned to more than one component position, and a limited supply of components is available for assignment. The optimal solution is found by ranking the reliabilities of the components of each type and searching over these ranks in the orders specified in this paper.

Unclassified

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
System Reliability						
Optimal Assignment						
Coherent Structures						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ______________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ______________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ______________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.